CONSTITUENT QUARK MODEL CALCULATION FOR A POSSIBLE $J^P=0^-$, T=0 DIBARYON

Georg Wagner, L.Ya. Glozman*, A.J. Buchmann, Amand Faessler

(Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14,

D – 72076 Tübingen, Germany)

Abstract:

There exists experimental evidence that a dibaryon resonance d' with quantum numbers $J^P=0^-$, T=0 and mass 2065 MeV could be the origin of the narrow peak in the (π^+, π^-) double charge exchange cross–sections on nuclei. We investigate the six–quark system with these quantum–numbers within the constituent quark model, with linear confinement, effective one–gluon exchange at short range and chiral interactions between quarks (π– and σ–exchange). We classify all possible six quark states with $J^P=0^-$, T=0, and with N=1 and N=3 harmonic oscillator excitations, using different reduction chains. The six–quark Hamiltonian is diagonalized in the basis including the unique N=1 state and the 10 most important states from the N=3 shell. We find, that with most of the possible sets of parameters, the mass of such a "dibaryon" lies above the N(939)+N*(1535) threshold. The only possibility to describe the supposed d'(2065) in the present context is to reduce the confinement strength to very small values, however at the expense of describing the negative parity resonances N*. We also analyze the $J^P=0^-$, T=2, N=1 six–quark state.

*Alexander von Humboldt Fellow, on leave from Alma–Ata Power Engineering Institute, Kosmonavtov 126, Alma–Ata, Kazakhstan.

1 Introduction

Believing QCD to be the theory of strong interactions, we consider hadronic systems as built up by quarks and gluons. With the exception of lattice calculations, which start directly from the underlying current quark and gluon fields, the descriptions of baryons make use of some effective degrees of freedom of nonperturbative QCD, such as constituent quarks [1], chiral fields [2], etc. In many–baryon systems, like atomic nuclei, the relevant degrees of freedom are baryons and mesons (except for perhaps some special cases [3], where the quark degrees of freedom must be taken into account explicitly).

However, in principle nothing forbids the existence of objects with baryon number bigger than one, which have some QCD motivated origin and could be considered as built up with quarks and gluons rather than baryons and mesons. Such objects would have, in contrast to atomic nuclei, quite a small size, of order of 1 fm.

This idea was very popular about 10–15 years ago and a lot of efforts were made to find such objects, both theoretically and experimentally (see, for example, refs.[4]–[10] and references therein). However, an experimental search for dibaryons in the NN–system has not been successful until now.

The reason for this seems to be quite clear. The only demand for the color part of the six–quark wave function of a hypothetical dibaryon is that it is a color singlet, i.e. it has a symmetry

$$[2^3]_C \equiv \young(\hfil\hfil,\hfil\hfil,\hfil\hfil)_C \quad . \tag{1}$$

This singlet $SU(3)_C$ representation contains in its Clebsch–Gordan expansion the state

$$[1^3]_C \otimes [1^3]_C \equiv \young(\hfil,\hfil,\hfil)_C \otimes \young(\hfil,\hfil,\hfil)_C \tag{2}$$

(Here and in the following, "×" denotes an inner product, whereas "⊗" denotes an outer one). But in this case the strong confinement forces $\sim \lambda_i^a \cdot \lambda_j^a$ are absent between two color singlet objects: With the usual normalization of the Gell–Mann

1

matrices $\vec{\lambda}^2 \equiv \lambda^a \lambda^a = \frac{16}{3}$ (summation over double indices), one finds

$$\langle \square_C \otimes \square_C \,|\, \lambda_i^a \cdot \lambda_j^a \,|\, \square_C \otimes \square_C \rangle = 0 \qquad , \qquad \begin{cases} i \subset \{1,2,3\} \\ j \subset \{4,5,6\} \end{cases} . \tag{3}$$

So, if two baryons are very close to each other and form a 6q–system, the confinement forces will not prevent such a system from a decay into baryon–baryon channels. This means, that the lifetime for such a system would be extremely small and the corresponding large width would thus not allow to treat such a system as a dibaryon. The only possibility for the dibaryon in this case would be if there were some forces (like $\lambda_i^a \cdot \lambda_j^a \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$, $\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$, etc.) which bind the six–quark system below the corresponding baryon–baryon threshold for a strong decay. This argument stimulated for example the search for an H–particle [4, 9, 10].

On the other hand, there seems to be some recent experimental evidence for a negative parity resonance in the πNN–system with the quantum numbers $\mathrm{J}^P = 0^-$, T=0 [11], for which the notation d' has been introduced. The pionic double charge exchange reaction (DCX) on nuclei

$$^A Z\,(\pi^+, \pi^-)\,^A Z + 2 \tag{4}$$

exhibits, independently of the nuclear target, a very narrow peak near the pion kinetic energy $T_\pi = 50$ MeV. Its position as well as its width are practically identical for the available world DCX–data on light and medium nuclei such as ^{12}C, ^{14}C, ^{18}O, ^{44}Ca, ^{48}Ca. Only its amplitude depends on the considered nucleus. This suggests, that it can only be connected to some elementary process.

Due to charge conservation this elementary process involves at least two nucleons within the nucleus. The hypothesis, that this peak is due to a dibaryon resonance d' in the πNN channel with quantum numbers $\mathrm{J}^P = 0^-$, T=0 and mass 2065 MeV [11], allows to describe all available data. This dibaryon cannot decay into the nucleon–nucleon channel due to the Pauli principle and its mass is below any baryon–baryon $(NN^*, N^*N^*, ...)$ threshold. These peculiarities explain the very small width of only $\Gamma \sim 5$ MeV of the d' dibaryon.

These circumstances motivated our detailed study of the six–quark system with the quantum numbers $J^P=0^-$, T=0 within a constituent quark model [3, 12, 13, 14, 15] with and without chiral interactions between constituent quarks.

Dibaryons with quantum numbers $J^P=0^-$, T=0 were already studied in the framework of string–like (deformed) bag models and were calculated as strings with q^4 and q^2 colored quark clusters at the ends [5, 8]. An essential shortcoming of this model is the lack of antisymmetrization between quarks belonging to different clusters. Antisymmetrization can be neglected only for well separated clusters. However, in the $q^4 - q^2$ system the color–exchange (confining) forces between quarks do not allow large separations, and the typical size of such a system is expected to be of order 1 fm. We know from experience in the NN–system that the Pauli principle on the quark level plays a decisive role at such distances [3, 12, 13].

The other drawback of the bag–model in the six–quark system is that the boundary conditions (which simulate the quark confinement) prevent such a system from the color–singlet 3q – color–singlet 3q clusterization, which is the most important phenomenon in 3n–quark ($n > 1$) systems. If such a model were correct, a large number of dibaryons should be observed. From this point of view, a potential model, where quark confinement is approximated by two–body q–q forces $\sim \lambda_i^a \cdot \lambda_j^a$, is more satisfactory, although one has to take care of the long–range Van–der–Waals forces in this case, when describing the NN–system [16].

In this work we study the six–quark system with the quantum numbers $J^P=0^-$, T=0 within the constituent quark model [3, 12, 13, 14, 15] taking into account properly the Pauli principle throughout the whole calculation. We investigate both possibilities; with a chiral field (pions and sigma mesons) coupled directly to constituent quarks, and without such a chiral field. In the latter case, quarks interact only through color–exchange potentials. In our previous communication [17], only the lowest shell–model state s^5p was used. In that paper, we did not care about a correct description of the one–quantum nucleon excitations N*(1535) ($J^P=\frac{1}{2}^-$, T=$\frac{1}{2}$). However, this is quite important, since the dibaryon mass should be compared with

3

the N(939)+N*(1535) threshold. Here we present a more complete calculation taking into account configuration mixing, including the 10 presumably most important excited states. We find that with all sets of parameters, that describe rather well the lowest baryon mass spectrum, the mass of the $J^P=0^-$, T=0 6q–state lies around 100 – 200 MeV above the NN*–threshold. However, including more excited states in our basis, this mass could come down below the threshold by some 10 MeV's. If it is so, there really could exist a dibaryon with these quantum numbers. On the other hand, its mass, at least within our model, is still higher than the experimentally observed peak suggests. Assuming that the confinement potential in a six quark system is weaker than the usual choice in three quark systems (This would still describe the nucleon and the Δ, but not the negative parity N* resonances.), yields a dibaryon with a mass close to 2065 MeV and an oscillator length $b_6 \simeq 1.25$ fm.

It seems that this 6q–system is very sensitive to the explicit form of the confinement mechanism, or, in other words, the extension of the baryon confinement to multi–quark confinement might not be too straightforward. In this sense, the confirmation of the d' resonance could give valuable information to our understanding of confinement.

The structure of the paper is the following: The effective Hamiltonian for the dibaryon is presented in section 2, and some short motivation will be given for its different ingredients. Fitting the parameters to the spectrum of light baryons is the commonly accepted procedure to fix the effective Hamiltonian. In section 3, we present the Translationally Invariant Shell–Model (TISM) basis. We classify the basis states for the 6q–system with $J^P=0^-$, T=0, and review some important points concerning the fractional parentage technique for the wavefunction. We present the results for the dibaryon, first excluding the mixing of excited states (cf. [17]), and then including the ten presumably most important excited states. A short discussion of a possible $J^P=0^-$, T=2 six–quark state is also added. Section 7 gives a conclusion of the present work. All necessary analytical expressions are reserved for the appendices.

2 The effective Hamiltonian

One now generally assumes, that the constituent quarks are quasiparticles with a complicated structure and dynamical (q–dependent) mass [18]–[21]. The current quarks of the underlying QCD acquire the constituent mass due to the spontaneous breaking of the chiral symmetry of the QCD Lagrangian, which is nearly exact in the $SU(2)$–flavour sector.

This chiral symmetry breaking is caused probably by the instanton structure of the QCD vacuum [19, 20] and is characterized by the corresponding order parameter – the non–zero quark condensate – and by the Goldstone exitations – the pions. All these features are also reproduced in the Nambu–Jona-Lasinio model (see recent review [21] and references therein), which can be considered as an effective approximation to the underlying QCD in the low–energy domain.

So, the low–momentum Lagrangian must contain the constituent quarks and chiral fields as effective degrees of freedom. But it must also contain a color–exchange interaction between the constituent quarks, since the latter are coloured objects. Among these effective interactions we should include a confinement force (whose real nature is not known), and also a short–range (i.e. at distances less than the chiral symmetry breaking scale) effective gluon–exchange between quarks.

There is a very close analogy to this picture coming from solid state physics. The correct effective degrees of freedom for an explanation of thermo– and/or electric properties of metals are not light elementary electrons and ions, but heavy electrons with effective mass m^* and phonons. Phonons are Goldstone excitations and appear as a result of the breaking of translational invariance in the lattice of ions. This spontaneous breaking is also characterized by the effective electron mass m^* (microscopically, this effective mass arises due to very complicated interactions between the elementary electrons and the lattice). There are also some residual interactions such as electron–electron, electron–phonon, etc.

After these quite general remarks and motivation for our chosen quark picture,

let us now turn to the concrete description of the effective Hamiltonian.

The chiral sector: Pion– and Sigma–exchange

The chiral–invariant interaction Lagrangian of the σ–model type [21, 22, 14], which describes in our case an interaction of the constituent quark field Ψ (spinor in $SU(2)_{flavour}$–space) and the chiral fields $\boldsymbol{\pi}$ and σ, is

$$\mathcal{L}_{int} = -g\overline{\Psi}(\sigma + i\gamma_5\boldsymbol{\pi}\cdot\boldsymbol{\tau})\Psi. \tag{5}$$

The constituent quarks have a complicated structure which can be parametrized by a formfactor in the quark–chiral field vertex. For that, we use the substitution

$$g \longrightarrow g\left(\frac{\Lambda^2}{\Lambda^2 + \mathbf{k}^2}\right)^{1/2} \tag{6}$$

where Λ characterizes the chiral symmetry spontaneous breaking scale, and must be of the order of the instanton size in the QCD vacuum [20], i.e. $\Lambda \simeq (0.7 - 1.5)\text{GeV}$.

This cut–off contributes to the pion–nucleon interaction radius as

$$\langle r^2 \rangle_{\pi N} = b^2 + \frac{3}{\Lambda^2} \qquad , \tag{7}$$

where b is the quark core size of the nucleon [15]. The larger the cut–off Λ, the more pointlike behaves the pion–quark coupling. Intuitively, one could think of this cut–off as an effective size of the constituent quarks $\langle r_q^2 \rangle = \frac{3}{\Lambda^2}$. For example Λ=4.2 fm^{-1} would correspond to a size of $\sqrt{\langle r_q^2 \rangle} \simeq 0.4$ fm. On the other hand, the cut–off formfactor behaviour in eq.(6) can be related to the q^2–dependence of the dynamical constituent quark mass [14].

The sigma mass m_σ is fixed by the relation $m_\sigma \simeq 2m_q$, where $m_q \simeq \frac{1}{3}m_N$ is the constituent quark mass. On the other side, the relation

$$m_\sigma^2 = 4m_q^2 + m_\pi^2$$

appears as a result of the bosonization of the Nambu and Jona-Lasinio Lagrangian.

The pion–quark coupling constant g is determined by the well known $g_{\pi N}$ coupling constant $g = \frac{3}{5}\frac{m_q}{m_N}g_{\pi N}$ $(g_{\pi N} = 13.36)$, where the coefficient $\frac{3}{5}$ comes from the

spin–isospin matrix element when we consider the πN interaction as the interaction between the pion and the 3 constituent quarks.

Our notations for the coupling constants are

$$f_{\pi q} = \frac{m_\pi}{2m_q} \cdot g \quad ; \quad g_{\sigma q} = \frac{m_\sigma}{2m_q} \cdot g \simeq g \quad . \tag{8}$$

Fourier–transforming the static pion propagator,

$$\frac{1}{(2\pi)^3} \int \exp(i\vec{q} \cdot \vec{r}) \frac{d^3\vec{q}}{\mu^2 + \vec{q}^2} = \frac{1}{4\pi} \frac{\exp(-\mu r)}{r} \tag{9}$$

we arrive at the usual Yukawa–like potential for the isovector pion (and similarily for the scalar–isoscalar sigma, which is not sensitive to any spin–isospin quantum-numbers of the quarks, but only to their orbital distribution) :

$$V_{ij}^\pi(\mathbf{r}) = \frac{\Lambda^2}{\Lambda^2 - m_\pi^2} \frac{f_{\pi q}^2}{4\pi} \frac{\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j}{3} \Big((\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j V_C(m_\pi r) + \hat{S}_{ij} V_T(m_\pi r)) - \frac{\Lambda^2}{m_\pi^2}(m_\pi \to \Lambda) \Big) \tag{10}$$

$$V_{ij}^\sigma(r) = -\frac{\Lambda^2}{\Lambda^2 - m_\sigma^2} \frac{g^2}{4\pi} \Big(V_C(m_\sigma r) - V_C(\Lambda r) \Big) , \tag{11}$$

$$V_C(mr) = \frac{\exp(-mr)}{r} \quad ; \quad V_T(mr) = \Big(1 + \frac{3}{mr} + \frac{3}{m^2 r^2} \Big) \frac{\exp(-mr)}{r} ; \tag{12}$$

$$\hat{S}_{ij} = \Big(\frac{3\boldsymbol{\sigma}_i \cdot \mathbf{r} \ \boldsymbol{\sigma}_j \cdot \mathbf{r}}{r^2} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \Big) \tag{13}$$

The Color–part: Confinement and Gluon exchange

We take the confinement potential, which mainly determines the medium– and long–range (on the quark scale) phenomena, to be linear

$$V_{ij}^{Conf}(r = |\mathbf{r_i} - \mathbf{r_j}|) = -\frac{\lambda_i^a \cdot \lambda_j^a}{4} (a_c r - C) \tag{14}$$

The effective "one–gluon exchange" potential, which is responsible for the very short–range phenomena and motivated in its form from the charmonium spectroscopy [23], contains essentially five terms:

$$V_{ij}^{OGE}(\mathbf{r} = \mathbf{r_i} - \mathbf{r_j}) = \frac{\alpha_s}{4} \lambda_i^a \cdot \lambda_j^a \Big\{ \frac{1}{r} - \frac{\pi}{m_q^2} \Big(1 + \frac{2}{3} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \Big) \delta(\mathbf{r}) - $$
$$- \frac{1}{4m_q^2 r^3} \cdot \hat{S}_{ij} - \frac{3}{m_q^2 r^3} \mathbf{r} \times \mathbf{p}_{ij} \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \Big\} \tag{15}$$

the so–called color–coulomb (CC $\simeq \lambda_i^a \cdot \lambda_j^a \frac{1}{r}$), the contact color–electric (CE $\simeq$ $\lambda_i^a \cdot \lambda_j^a \, \delta(\vec{r})$) term, the contact color–magnetic (CM $\simeq \lambda_i^a \cdot \lambda_j^a \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \, \delta(\vec{r})$) term, the tensor term proportional $\hat{S}_{ij}$ and the Galilei–invariant spin–orbit term.

In our present study, we drop the spin–orbit term. As it is well known, there is practically no room for the spin–orbit interaction when describing the baryon spectrum. It is well seen, for example, from the small mass–splitting of the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ resonances N*(1535) and N*(1520). Usually it is assumed, that the Galilei–invariant spin–orbit forces from the OGE–potential are compensated in the baryons by the spin–orbit forces from other sources, e.g. the Thomas term coming from the confinement [1] or the σ–exchange [24].

Parameter fitting to light baryons

The constituent quarks, π– and σ–fields, and the color–exchange forces are some effective degrees of freedom of nonperturbative QCD. They do not follow from first principles in a compelling and unique way, but are only supposed to simulate the dominant features of QCD in the low energy domain. The strong coupling constant α_s for example, is not the momentum–dependent running coupling constant of QCD, but an effective parameter, which has to be fixed to some observable of low energy QCD to define our Hamiltonian.

A possible way to fit the 5 parameters of our final Hamiltonian of eq.(16), $\alpha_s, a_c, C, \Lambda$ and m_q, is to describe the baryon spectrum, or more precisely the light baryons such as the nucleon, the Δ–resonance, and the $J^P=\frac{1}{2}^-$,T$=\frac{1}{2}$ one–quantum excitation N*(1535) (the reason, why one should fit our parameters first of all to the N*(1535), but not to the other excited states, will be seen from considerations in section 4). For the N and Δ, we use the simple s^3 harmonic oscillator functions (without the center–of–mass oscillations: see states $|N_0\rangle$ and $|\Delta_0\rangle$ in appendix A and in refs.[3, 15]). The analytical expressions for the mass–formulae $m_N = h_{00}^N$ and $m_\Delta = h_{00}^\Delta$ can be found in appendix A.

To suppress the mixing with the two– and more–quantum harmonic oscillator

states in the N and the Δ, we impose the nucleon stability condition [13] $\frac{\partial m_N}{\partial b_N} = 0$, where b_N is the harmonic oscillator parameter in the 3–quark system (which coincides with the mean–square quark core radius of the N and the Δ).

For the N*(1535), we diagonalize analytically our Hamiltonian in the space including the two possible negative parity one–quantum states $|N_1^{(-)}\rangle$ and $|N_2^{(-)}\rangle$ in notations of the second paper in ref.[3] (All the necessary wave functions can be found there), and fit the smaller eigenvalue to the N*(1535) mass. The corresponding mass–matrix can also be found in appendix A[1].

Thus, for the six free parameters, $\alpha_s, a_c, C, \Lambda, b_N, m_q$, we have now four constraints. But there is not to much freedom because one has the following additional, approximate constraints:

i) the parameter Λ is fixed by the scale of the chiral symmetry spontaneous breaking, $\Lambda \simeq 0.7 - 1.5$ GeV,

ii) the quark core radius of the nucleon should be in the region $b_N \simeq 0.45 - 0.6$ fm,

iii) the constituent quark mass should be $m_q \simeq 200 - 400$ MeV.

In table 1, we show possible sets of parameters. The sets I–III correspond to the inclusion of all types of the q–q forces, while in set IV, there are no chiral fields inside the baryons, i.e. no $\pi-$ and σ–exchange between quarks. This selection does not give a systematic study of how the results depend on the chosen parameters, but is meant to show some qualitative features of our results.

Increasing b_N up to $b_N = 0.6$ fm in the above discussed parameter fitting scheme, we get a too small constituent quark mass, $m_q \simeq 170$MeV.

We also show in this table the model mass for the two–quantum positive parity $J^P = \frac{1}{2}^+, T = \frac{1}{2}$ resonance. This mass is calculated by diagonalisation of our Hamiltonian with already fixed parameters in the space including two possible two–quantum excitations $|N_1\rangle$ and $|N_2\rangle$ with L=0 (see ref.[3] and appendix A).

We see that this mass is essentially higher than the mass of the Roper resonance.

_____

[1]In this basis, the 2nd eigenvalue can be compared to the next one–quantum excitation with $J^P = \frac{1}{2}^-, T = \frac{1}{2}$, the N*(1650). We give this result for illustration in table 1.

The Roper resonance is a longstanding problem in the description of baryons within the constituent quark model. We would like to remark at this point, that there are some new ideas [25] for the CQM, that seem to overcome this problem.

Set V finally gives a set of parameters, determined from the following fitting scheme, which will give the dibaryon results closest to the "experimental value". To obtain these parameters, we fix as input the constituent quark mass m_q, the cut–off Λ and the nucleon hadronic size b_N to determine α_s, a_c, C by the Δ–N mass splitting, the nucleon stability condition, and the nucleon mass respectively. In the table we see the small confinement strength $a_c \simeq 25$ MeV/fm, which goes together with a bigger quark core size of the nucleons b_N=0.6 fm. These parameters emerge also naturally, if the nucleon quark core size b_N is determined by the nucleon stability condition. A similar parameter set is for example used to describe the nucleon magnetic moments in [15]. However, it is not possible to simultaneously describe the one–quantum excitations with these parameters.

So, with fixed parameters, the effective 6–quark Hamiltonian is given by

$$H^{(6q)} = \sum_{i=1}^{6} \left(m_q + \frac{\mathbf{p}_i^2}{2m_q} \right) - \frac{\mathbf{P}_{cm}^2}{12m_q} + \sum_{i<j}^{6} V_{ij}^{Int} \qquad , \qquad (16)$$

where

$$V^{Int} = V^{Conf} + V^{OGE} + V^{\pi} + V^{\sigma} \qquad . \qquad (17)$$

This Hamiltonian contains interactions of two particles only, so the two–particle fractional parentage coefficients (fpc), which will be reviewed below, are particularly well suited to evaluate matrix elements for two–body interactions of the above type.

3 The Translationally Invariant Shell Model basis (TISM)

The harmonic–oscillator basis is highly successful in describing the baryon spectrum. Let us recall, why this is so. The reason is, of course, that the quark–quark harmonic

oscillator attraction simulates well the quark–quark confinement forces in an 3–quark color–singlet object. Indeed, the color–singlet 3–quark wave function

$$[1^3]_C \equiv \begin{array}{c}\square\\\square\\\square\end{array}_C \tag{18}$$

is antisymmetric in color space in any quark pair. Since

$$\langle \,\begin{array}{c}\boxed{i}\\\boxed{j}\end{array}_C |\lambda_i^a \cdot \lambda_j^a| \begin{array}{c}\boxed{i}\\\boxed{j}\end{array}_C \rangle = -\frac{8}{3} \quad , \tag{19}$$

the confinement potential of eq.(14) is attractive in any quark–quark pair. Of course, historically, the confinement potential was just constructed as to provide an attraction.

In the six–quark system, the situation is very different. Here the color part is characterized by the mixed permutational symmetry

$$[2^3]_C \equiv \begin{array}{cc}\square&\square\\\square&\square\\\square&\square\end{array}_C . \tag{20}$$

It means, that there are both antisymmetrical and symmetrical pairs. But in the symmetrical ones, instead of attraction, the potential (14) leads to repulsion, since

$$\langle \,\boxed{i\,j}_C |\lambda_i^a \cdot \lambda_j^a| \boxed{i\,j}_C \rangle = \frac{4}{3} \quad . \tag{21}$$

So, the confinement forces (14) try to build the six–quark system in such a way as to divide it into two color–singlet 3–quark clusters.

It is not possible to simulate this behaviour with only one harmonic oscillator six–quark configuration. In the harmonic oscillator wave function we have an attraction in each quark–quark pair[2]. However, one can use the 6q harmonic oscillator as a basis to diagonalize the Hamiltonian eqs.(16),(17). In this case, it is intuitively clear, that the confinement force will mix the different 6q–harmonic oscillator states in such a way as to provide in the final six–quark wave function a color–singlet – color–singlet clusterization. If the baryon–baryon asymptotics with well separated baryons (e.g. for the deuteron) exists in some six–quark system, the harmonic oscillator basis is

---

[2]Due to the same reason, the bag model for a six–quark system is conceptually not correct.

not efficient to describe this system at all range, since we would need a tremendous number of highly excited harmonic oscillator configurations in this case[3].

In the 6q–system with quantum numbers $J^P=0^-$, T=0, the nucleon–nucleon clusterization is suppressed due to the Pauli principle on the nucleon level. On the other hand, if there is a deeply bound state with respect to the $N(939)+N^*(1535)$ threshold, the harmonic oscillator basis could be quite successful in describing such a system.

The shell–model basis allows, as we will see in more detail in the following, to respect the Pauli principle, i.e. the antisymmetrization of the total wavefunction, at any level of the calculation, so that effects arising due to the Pauli–principle are naturally included in the formalism. This is very important for interquark distances around or smaller than 1 fm, which are expected for a dibaryon. The importance of the Pauli principle is well known from the NN–system. There, the short range repulsion at distances smaller 1 fm is essentially due to quark exchanges, caused by the Pauli principle on the quark level [12].

Furthermore, another advantage of this Hamiltonian lies in the fact, that the center–of–mass (cm) motion is removed properly, so as to exclude spurious states from the harmonic oscillator basis.

The price we pay for the exact inclusion of the Pauli principle and unambigious exclusion of spurious states in our basis, is of course the uncertainties induced by the non–relativistic description and by the effective nature of our Hamiltonian, which is not derived from first principles, but is assumed to simulate the dominant features of low–energy QCD.

We remind the reader that the harmonic oscillator six–particle states are exact

---

[3]But if the six–quark harmonic oscillator states at small distances are combined with the corresponding clusterized configuration like $\hat{\mathcal{A}}\{B_1(1,2,3)B_2(4,5,6)\,\chi(\vec{r})\}$ at medium and large range in one variational task, we do not need much harmonic oscillator states at small distances. Such a basis is essentially more flexible than the one–channel RGM basis. This variational program was realized in part in the investigation of the NN problem in refs.[14, 26].

solutions of the Translationally Invariant Shell Model (TISM) Hamiltonian, ref.[27].

$$
\begin{aligned}
H^{(A=6)} &= \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_q} - \frac{\mathbf{P}_{cm}^2}{2Am_q} + \frac{1}{2} m_q \omega^2 \sum_{i=1}^{A} (\mathbf{r_i} - \mathbf{R_{cm}})^2 \\
&= \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_q} - \frac{\mathbf{P}_{cm}^2}{2Am_q} + \frac{m_q \omega^2}{2A} \sum_{i<j=1}^{A} (\mathbf{r_i} - \mathbf{r_j})^2 \quad .
\end{aligned}
\tag{22}
$$

The harmonic oscillator–eigenstates of this Hamiltonian

$$
|\text{h.o.} - \text{state}\rangle = |A = 6\,, N\,[f]_X\,(\lambda\mu), L, S, T\,, \alpha\rangle
\tag{23}
$$

are classified (cf. ref.[27]) by the following set of quantum numbers; N indicates the number of internal excitation quanta, the Young–pattern $[f]_X$ determines the spatial permutational symmetry, the Elliot symbol $(\lambda\mu)$ gives the $SU(3)$ harmonic oscillator multiplet, L, S, T are the total orbital angular momentum, total spin and total isospin. The total spin S is uniquely connected with the Young pattern $[f]_S$ for the $SU(2)_S$ representation by $S = (f_1 - f_2)/2$ where f_1 and f_2 are the first and second rows in the Young pattern $[f]_S$. The same applies to the total isospin T. The unambigious definition of a h.o.–state requires, as will be shown in a moment, additional quantum numbers, here collectively denoted by α.

In conventional nuclear physics (dealing with nucleons), due to the Pauli principle, the spatial Young pattern $[f]_X$ must be conjugate to the $SU(4)_{ST}$ Young pattern $[f]_{ST}$ in order to provide the total antisymmetry (i.e. $[f]_X = [\tilde{f}]_{ST}$, where the tilde implies the Young pattern reversed with respect to the main diagonal).

In the quark model (with two flavours), due to the additional degree of freedom *color*, the classification is more complicated. For a six–quark system, the full permutational color symmetry is described by the $[2^3]_C$ color Young pattern ($SU(3)_C$ color singlet). To identify unambigiously the six–quark state we have to use additional quantum numbers, because the inner product of the $SU(3)_C$ color–singlet representation and the $SU(2)_S$ spin representation (or the $SU(2)_T$) contains in general more than one representation of the $SU(6)_{CS}$ group [4, 28, 29], $SU(6)_{CS} \supset SU(3)_C \times SU(2)_S$ (or $SU(6)_{CT}$). The spatial symmetry $[f]_X$ determines

uniquely only the $SU(12)_{CST}$ permutational symmetry, $[f]_X = [\tilde{f}]_{CST}$; so we need in addition $[f]_{CS}$ or $[f]_{CT}$. Other reduction chains are also possible, for example with the intermediate spin–isospin $SU(4)_{ST}$ symmetry [30], or with the intermediate spin–spatial symmetry [31], etc. All these reduction chains are equivalent, and the choice is determined by convenience (for different types of quark–quark forces different reduction chains are convenient).

In our preliminary communication [17], we have used the $SU(6)_{CS}$ and $SU(4)_{ST}$ symmetries to classify all possible states. In the present paper the choice of the intermediate spin–spatial permutational symmetry $[f]_{XS}$ is entirely motivated by the available tables of two–particle fractional parentage coefficients in refs.[31].

Classification of the states with $J^P = 0^-$, T=0.

We get the negative parity only if our states contain an odd number of harmonic oscillator excitations, i.e. the possible number of excitation quanta N is restricted to

$$N = 1, 3, 5, ... \tag{24}$$

There is only one state with N=1, compatible with $J^P = 0^-$, T=0:

$$|N = 1, [51]_X, (\lambda\mu) = (10), L = 1, S = 1, T = 0, [42]_{XS}\rangle . \tag{25}$$

In this case, all possible intermediate permutational symmetries (within different reduction chains) are determined simultaneously and uniquely

$$[321]_{CS} , [321]_{ST} , [2211]_{CT} , [321]_{XT} , [42]_{XS} , [321]_{XC} . \tag{26}$$

This harmonic–oscillator state (i.e. TISM–state) is uniquely connected with the harmonic–oscillator shell model state $s^5 p$ as follows:

$$|s^5 p\, [51]_X\, (10)L = 1\rangle = \Phi_{000}(\vec{\mathbf{R}}_{cm}) \cdot |N = 1\, [51]_X\, (10)L = 1\rangle \tag{27}$$

The shell model state $s^5 p$ with the spatial symmetry $[6]_X$ is a spurious one, since

$$|s^5 p\, [6]_X\, (10)L = 1\rangle = \Phi_{111}(\vec{\mathbf{R}}_{cm}) \cdot |N = 0\, [6]_X\, (00)L = 0\rangle \tag{28}$$

14

There are a lot of different states with three internal excitation quanta.

The full classification of the orbital parts with N=3 and the spatial symmetries $[42]_X, [411]_X, [33]_X, [321]_X$, and $[3111]_X$ was done in Ref.[27]. In addition we have also the states with $[51]_X$ spatial symmetries (both $[6]_X$ and $[51]_X$ are also allowed in the quark–model, but the state with N=3, $[6]_X$ is a spurious one).

However, not all of these orbital states are compatible with the color, spin and isospin quantum numbers of the d'. In table 2 we present all possible configurations with N=3, compatible with the quantum numbers $J^P=0^-$, T=0.

As mentioned above, all different reduction chains are equivalent. For example, for the two reduction chains

$$SU(12)_{CST} \quad \supset \quad SU(6)_{CS} \times SU(2)_T \supset SU(3)_C \times SU(2)_S \times SU(2)_T \quad , \quad (29)$$

$$SU(12)_{CST} \quad \supset \quad SU(3)_C \times SU(4)_{ST} \supset SU(3)_C \times SU(2)_S \times SU(2)_T \quad , \quad (30)$$

the intermediate $SU(6)_{CS}$ or $SU(4)_{ST}$ symmetries are needed respectively to define the shell–model states uniquely. They are given in line 4 and 5 of table 2 (see for more details ref.[17]).

The reduction chain we finally adopt in our calculation, decouples color and isospin from the remaining spin–orbital symmetry.

$$[1^6]_{XCST} \quad \supset \quad [33]_T \times [f'']_{XCS} \supset [33]_T \times [222]_C \times [f]_{XS}$$
$$\supset \quad [33]_T \times [222]_C \times [f]_S \times [f']_X \qquad (31)$$

In the sixth line of table 2, we give the intermediate $[f]_{XS}$ symmetry, needed in this classification scheme.

The rules for inner products of Young patterns for certain permutational symmetries can lead to the situation, that a certain product symmetry occurs more than once. In these cases we need an additional quantum number (in table 2, these quantum numbers are indicated by subscripts) to distinguish these different states.

The last line of table 2 gives the number of states in each column, all in all, we find 31 TISM–states with N=3 and the quantum numbers of the d'. Even if it were

possible to include all these 31 excited states in our calculation (this would give a 32×32 matrix problem), this would not be a complete basis, since we are not taking higher excitations with $N = 5, 7, ...$ into account. For a qualitative estimate of the effect of configuration mixing with the N=3 states, we choose the 10 states in the third row of table 2, characterized by

$$N = 3, \ [42]_X, \ L = 1, S = 1, T = 0 \quad , \tag{32}$$

in order to build a basis of 11 states for our calculation. The notation for any of these ten states is analogous to eq.(25) and can be easily written down with the help of table 2.

The reason why we choose these states is, that there are the two most "symmetrical" (i.e. the longest) Young patterns $[f]_{CS}$ and $[f]_{ST}$ among these states. We remind the reader, that the more symmetrical pairs in color–spin–space we have, the more attraction in the six–quark system we get, due to the color–magnetic $\lambda_i^a \cdot \lambda_j^a \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$ forces in eq.(15).

This is well seen from the matrix elements

$$\langle [f_{ij}]_C \times [f_{ij}]_S| \ \lambda_i^a \cdot \lambda_j^a \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \ |[f_{ij}]_C \times [f_{ij}]_S\rangle = \begin{cases} \frac{4}{3}, & \text{if } [2]_C, \ [2]_S, \ [2]_{CS} \\ 8, & \text{if } [11]_C, \ [11]_S, \ [2]_{CS} \\ -4, & \text{if } [2]_C, \ [11]_S, \ [11]_{CS} \\ \frac{-8}{3}, & \text{if } [11]_C, \ [2]_S, \ [11]_{CS} \end{cases} \tag{33}$$

The same situation takes place for the pion–exchange forces (10). Here, one has

$$\langle [f_{ij}]_S \times [f_{ij}]_T| \ \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \ |[f_{ij}]_S \times [f_{ij}]_T\rangle = \begin{cases} 1, & \text{if } [2]_S, \ [2]_T, \ [2]_{ST} \\ 9, & \text{if } [11]_S, \ [11]_T, \ [2]_{ST} \\ -3, & \text{if } [11]_S, \ [2]_T, \ [11]_{ST} \\ -3, & \text{if } [2]_S, \ [11]_T, \ [11]_{ST} \end{cases} \tag{34}$$

So, at quark–quark distances larger than, for example, 1 fm, where the first term in eq.(10) dominates over the third one, one has an attraction due to central π– exchange forces in antisymmetrical ST–pairs. At small quark–quark distances (less than 1 fm), the cut–off in eq.(10) dominates, and one has an attraction in symmetrical ST–pairs. Since the characteristic size of our system is less than 1 fm, one has an attraction in symmetrical ST–pairs.

16

The fpc (fractional parentage coefficients) technique

Here, we cite briefly the definition and the application of the fractional parentage expansion (fpc), refering the reader to the corresponding literature [27]–[31] for more details.

Each six–quark harmonic oscillator configuration can be presented by means of the fractional parentage technique and the Talmi–Moshinsky transformation as a superposition of various four–quark × two–quark × relative motion components:

$$|N \ \alpha \ LSJT\rangle = \sum_{B_1, B_2, n, l} \Gamma^\alpha_{B_1, B_2, n, l} \Big\{ \phi_{B_1}(1, 2, 3, 4) \, \phi_{B_2}(5, 6) \, \phi_{nl}(\vec{r}) : LSJT \Big\} \quad (35)$$

Here, α stands for the necessary set of additional quantum numbers, including $[f]_X, (\lambda\mu), [f]_{XS}$, etc. to define unambigiously the six–quark state, and $\Gamma^\alpha_{B_1, B_2, n, l}$ is the so–called fractional parentage coefficient (fpc) in the TISM.

ϕ_{B_1} and ϕ_{B_2} are the TISM states for the first four particles and for the last two ones respectively. $\phi_{nl}(\vec{r})$ is the n–quantum harmonic oscillator function with

$$\vec{r} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4}{4} - \frac{\vec{r}_5 + \vec{r}_6}{2} \quad .$$

The summation in eq.(35) is carried out over all possible internal states of clusters B_1 and B_2 and their relative motion n, l, provided that

$$N_1 + N_2 + n = N \quad .$$

The specific feature of expansion (35) is, that the antisymmetric six–quark wave function is expanded into the sum of orthogonal, but not fully antisymmetric terms. Each term is antisymmetric only within the clusters B_1 and B_2.

By use of expansion (35), the interaction energy matrix elements of a pure two–body potential $H_{int} = \sum_{i<j}^N V_{ij}$ can directly be evaluated:

$$\langle \ N \ \alpha \ LSJT | \ H_{int} \ |N \ \alpha' \ L'S'JT\rangle = \frac{6(6-1)}{2} \times \quad (36)$$

$$\times \sum_{B_1, B_2, \tilde{B}_2, n, l} \Gamma^\alpha_{B_1, B_2, n, l} \Gamma^{\alpha'}_{B_1, \tilde{B}_2, n, l} \times \begin{pmatrix} \text{angular} \\ \text{momentum} \\ \text{recoupling} \\ \text{matrix} \end{pmatrix} \times \langle \phi_{B_2}(5, 6) | V_{56} | \phi_{\tilde{B}_2}(5, 6)\rangle$$

17

The expression (36) is written in symbolical form to avoid bulky $6j-$ and $9j$–symbols appearing due to the necessary recoupling of the angular momenta when calculating a contribution of the non–central forces. Here, a summation over all intermediate momenta is also assumed. For central forces, the recoupling matrix is absent.

It will not be our task here to rederive the rules of how to construct the two–particle fpc's for the 6–particle system. General considerations for the construction of fpc's can be found in various publications, as for example in [27]–[31] for the 6–quark system, and the fpc's (or more precisely the scalar factors) needed in our case are tabulated in [28, 29] and [31]. It is shown in these articles (and in the articles cited therein), that for a given reduction chain, the total fpc–coefficient factors out in a product of several scalar factors of the Clebsch–Gordan coefficients of the corresponding unitary groups, each one associated with a single step of the reduction.

In our case, the corresponding fpc are given as a product of scalar factors

$$\Gamma^{\alpha}_{B_1,B_2,n,l} = SF^U_{XSC\cdot T} \cdot SF^U_{XS\cdot C} \cdot SF^U_{X\cdot S} \cdot SF^U_C \cdot \Gamma_{TISM} \quad . \tag{37}$$

The first factor in eq.(37) is the weight factor and is determined only by the dimensions of the irreducible representations of the permutational group in isospin space for six particles (Young pattern $[f]_T$) and for the first four particles in eq.(35) (Young pattern $[f_1]_T$):

$$SF^U_{XSC\cdot T} = \sqrt{\frac{\dim[f_1]_T}{\dim[f]_T}} \quad . \tag{38}$$

The next two factors in eq.(37) can be extracted from the tables $2-5$ in ref.[31]. The color scalar factor (i.e. the scalar part of the $SU(3)_C$ Clebsch–Gordan coefficient in the reduction $SU(3)_C \supset O(3)_C$) can be found in table 1 of ref.[28].

Finally, the last factor in eq.(37) Γ_{TISM}, i.e. the orbital part of the fpc, can be found in table 2k of the first paper from [27]. This table contains all the necessary coefficients for N=3, $[f]_X = [42]_X$ six–particle harmonic oscillator configurations. For the lowest configuration, i.e. for $|N = 1\,[51]_X\,(\lambda\mu) = (10)\,L = 1, S = 1, T = 0\rangle$,

18

these coefficients are trivially calculated, and one finds:

$$\langle N\!=\!1[51]_X(10)\,L\!=\!1|N_1=0[4]_X(00)\,L_1=0\,;nl=00\,,N_2=1[11]_X\,L_2=1\rangle = -1$$

$$\langle N\!=\!1[51]_X(10)\,L\!=\!1|N_1=0[4]_X(00)\,L_1=0\,;nl=11\,,N_2=0[2]_X\,L_2=0\rangle = -1$$

$$\langle N\!=\!1[51]_X(10)\,L\!=\!1|N_1=1[31]_X(10)\,L_1=1\,;nl=00\,,N_2=0[2]_X\,L_2=0\rangle = 1$$

$$(39)$$

4 Results for the single N=1 configuration

As our full six–quark Hamiltonian of eq.(16) for the dibaryon comprises not only confinement, but also effective gluon–, pion– and sigma–exchange, it is very different from the harmonic oscillator Hamiltonian of eq.(22).

In our previous paper [17], we treated the difference between the effective Hamiltonian of eq.(16) and the harmonic–oscillator Hamiltonian of eq.(22) as a residual interaction. The corresponding diagonal matrix element for the lowest possible configuration, given by eq.(25)

$$M^{(N=1)}=\langle N=1,[51]_X(10)\mathrm{LSTJ}=1100|H^{(6q)}|N=1,[51]_X(10)\mathrm{LSTJ}=1100\rangle\,,$$

$$(40)$$

can be found in ref.[17] and also in appendix B.

As we already mentioned in the previous sections, the confinement forces of eq.(14) work towards the color–singlet + color–singlet clusterization in the 6q– system. In our case, that could be $NN^*_{\frac{1}{2}-}(1535)$ and $NN^*_{\frac{3}{2}-}(1520)$ clusterizations. We can exclude higher excited nucleon states or N^*N^* clusterizations, since their threshold energies would be quite high. However, the $NN^*_{\frac{1}{2}-}(1535)$ is highly preferable compared to the $NN^*_{\frac{3}{2}-}(1520)$. The reason is, that the latter cluster component in the $J^P=0^-$, T=0 six–quark system is not compatible with the lowest possible harmonic oscillator configuration of eq.(25). Indeed, in the $NN^*_{\frac{3}{2}-}(1520)$ system, the relative motion angular momentum must be $l=2$ to provide total angular momentum and parity $J^P=0^-$ of the six–quark state. Such an angular momentum l

needs a non–zero number of harmonic oscillator quanta, $n = 2, 4, \ldots$, corresponding to the relative motion. So, together with the one quantum from the internal N* excitation, the total number of harmonic oscillator quanta would be at least 3. The NN$^*_{\frac{1}{2}+}$(1440) component is also incompatible with the N=1 six–quark state. Thus, the "dynamical" threshold for a possible J^P=0$^-$, T=0 dibaryon is N(939)+N*(1535).

It is instructive to compare the excess energy over threshold in our case, i.e. $\delta M = M^{(N=1)} - (m_N + m_{N^*})$, with the corresponding value in the deuteron–like six–quark system J^P=1$^+$,T=0 ($^3S_1$ NN–wave), $\delta M_{s^6} = M_{s^6} - 2m_N$. In the latter case, as we know, there exists no compact 6q–state and the bound system (deuteron) consists of two weakly bound nucleons, which with overwhelming probability are far from each other.

We see from table 3 that δM is very similar to δM_{s^6}. This maybe a hint, that there could be a weakly bound state (below the N(939)+N*(1535) threshold) in the J^P=0$^-$,T=0 system. The reason is quite clear. There are more symmetrical pairs with CS–symmetry, $[321]_{CS}$, in the J^P=0$^-$,T=0 system, than with the $[2^3]_{CS}$ symmetry corresponding to the deuteron–like s^6 configuration. But the ST Young pattern for the J^P=1$^+$,T=0, s^6 configuration, $[33]_{ST}$, is more symmetrical than $[321]_{ST}$, inherent in the state (25) according to eq.(26). As a consequence the gain in energy coming from the color-magnetic forces (prefering a symmetric CS–configuration) is approximately compensated by the loss in energy arising from the chiral-exchange interaction (which prefers symmetrical ST–configurations). As a consequence, both systems have very similar excess energies.

In table 3, we have shown the excess δM calculated with the same harmonic oscillator parameter b_6 in the six–quark system as for the baryons $b_6 = b_N$. However, with fixed parameters of the quark–quark interactions of eq.(17), one should minimize the "mass" eq.(40) with respect to the harmonic oscillator parameter b_6 in the six–quark trial wave function. The corresponding value b_6 and the mass of the 6q–state $M^{T=0}_{N=1}$, are shown in the 3rd and 4th column of table 3. A more detailed discussion of these results for different parameter sets can be found in [17].

5   Configuration mixing results

Here, we report on the diagonalisation of the Hamiltonian (16) in the space including 11 harmonic oscillator configurations (the only state in the N=1 shell, and 10 states with a total number of three harmonic oscillator quanta excited: N=3,$[42]_X$, L,S,T=1,1,0). The quantum numbers and our labelling of these states, as used in the calculation, are given in table 4. All the needed matrix elements

$$H_{ij} \; = \; \langle \, i \, | \, H \, | \, j \, \rangle \qquad ; \qquad i,j = 1 \ldots 11 \tag{41}$$

can be found in appendix B.

It is important to mention the well known fact that a δ–type attractive two–body potential would result in a collapse in the three–body system, if a complete basis were used. Thus, the δ–type forces in eq.(15) have to be considered as an effective interaction, only valid for a given finite basis. We included the contact forces in perturbation theory and in the full diagonalisation, and obtained within 2 to 3 MeV the same masses for the calculated resonance J^P=0$^-$,T=0.

Since our basis is not complete, we improve its flexibility by using the harmonic oscillator parameter b_6 in the six–quark basis functions as a nonlinear variational parameter. More clearly, with fixed parameters of the quark–quark interactions, we minimize the lowest eigenvalue of (41) with respect to b_6. This means in particular, that the harmonic oscillator parameters (and thus the root mean square radius) of the baryons b_N and in the dibaryon b_6 need not to be the same.

Let us now discuss different contributions to the mass from the Hamiltonian (16). Since the diagonalisation of eq.(41) is a very nonlinear procedure, it is difficult to estimate the importances of the separate parts of the Hamiltonian (16). Nevertheless, diagonalising for example the confinement matrix (table B.1) alone, one recognizes that the mixing due to the confinement is essential. This result is independent of the chosen parameter set, since the parameter dependence factors out for the diagonalisation as is seen from eq.(B.3) in appendix B.

The same argument holds for the individual diagonalisation of the color–Coulomb interaction eq.(B.4). In this context, it is important to see, that large mixing amplitudes for the diagonalised system do not necessarily lead to a large gain in energy by the mixing calculation. For the confinement for example, the energy gain is rather important (around 100–150 MeV) whereas in the case of the color–Coulomb interaction, we gain only about 30 MeV.

One should mention here, that the effect of all forces, that involve only spatial and color degrees of freedom in their potentials, could be studied more easily in the basis with intermediate ST–symmetry. Here, these forces are diagonal with respect to the spin-isospin symmetry, and we have no mixing between states with different ST–symmetries for this part of the Hamiltonian. This means that only two of the 10 chosen states from the N=3 shell with $[321]_{ST}, (\lambda\mu) = (11)$ and $[321]_{ST}, (\lambda\mu) = (30)$ would be mixed in a ST–basis to the $N = 1, [321]_{ST}$ state by the confinement, color–Coulomb and σ–exchange forces. Using the ST reduction chain (30), only the color–magnetic, tensor and/or pion interactions would mix among states with different ST.

With all parts of the Hamiltonian (16), this analysis of the interplay of the different potential parts cannot be done anymore. Of course, the difference in kinetic energy from the N=1 and the N=3 shell of $\frac{1}{m_q b^2} \simeq 300 - 500$ MeV is the most obvious reason for the rather moderate gain in energy as seen from table 3. table 3 shows in its last 2 columns first the harmonic oscillator length b_6, for which the minimum of the mass is reached, and second the corresponding mass eigenvalue M for the chosen parameter set. This value should be compared (for the parameter sets I to IV) to the N(939)+N*(1535) threshold of 2474 MeV.

Comparing the results with and without configuration mixing, we see that the inclusion of the excited states shifts the mass down by about $100 - 150$ MeV. In the best case (for set II), the mixing result is 90 MeV above the N(939)N*(1535) threshold. The inclusion of higher configurations makes the 6q–system larger, and the confinement strength determines essentially the size of the dibaryon b_6. There-

22

fore, since parameter set V allows for a large dibaryon of $b_6 = 1.25$ fm, and since the kinetic energy is proportional $1/b^2$, we see qualitatively, why this set gives the lowest "mass" of the dibaryon in our basis. One can see in table 1 that this parameter set does not allow for a description of the excited baryon N*(1535). Thus, we conclude that it is difficult to describe the baryon spectrum and the dibaryon mass with the same confinement parameters.

6   The $J^P{=}0^-$, T=2 six–quark state as a candidate for the d'

In this section, we discuss shortly the question of a $J^P{=}0^-$, T=2 six–quark state, since from the very beginning, it is not clear which state, T=0 or T=2, is lower in mass [32]. Both possible states can not decay into NN–channels due to the Pauli–principle.

In our approach, following the classification (31) in section 3, we get two states with one excitation quantum N=1 and the quantum numbers $J^P{=}0^-$, T=2.

$$|1\rangle \;\; = \;\; |N{=}1, [51]_X, (10), L{=}1, S{=}1, T{=}2, [321]_{XS}\rangle \qquad (42)$$

$$|2\rangle \;\; = \;\; |N{=}1, [51]_X, (10), L{=}1, S{=}1, T{=}2, [42]_{XS}\rangle \qquad (43)$$

With the other possible reduction chains (29) and (30), we get for (30) the two states $[42]_{ST}$ and $[321]_{ST}$, and for (29) $[31^3]_{CS}$ and $[21^4]_{CS}$. The necessary diagonal matrix–elements for the states (42) and (43) are given in appendix C.

In columns 5 and 6 of table 3, we give the minimized expectation values (C.1) and (C.2) for the lowest possible states $|1\rangle$ and $|2\rangle$ in comparison with the result for the N=1, $J^P{=}0^-$, T=0 state. From table 3 we see, that within the constituent quark model both the $J^P{=}0^-$,T=0 and the $J^P{=}0^-$,T=2 state are nearly degenerate.

The reason for this degeneracy is clearly the competition between the color–magnetic part of the gluon–exchange and the central pion–exchange contribution. In analogy to the discussion in section 3, we see here, that the T=0 state with

its $[321]_{CS}$ symmetry provides more attraction from the color–magnetic interaction, than the less symmetric CS–symmetries of the T=2 states. The competing process is the pion–exchange part, which provides for the $[42]_{ST}$ symmetry of the T=2 state more gain in energy, than in the N=1, T=0, $[321]_{ST}$ state. So, at the end, the results depend very much on the given parameter set, or more precisely on the relative importance of the one–gluon– and the pion–exchange. Both six–quark systems are quite close in energy, and from the point of view of the constituent quark model neither the T=0 nor the T=2 state are energetically favoured.

7 Conclusion

We have investigated the six–quark system with the quantum numbers $J^P=0^-$, T=0 within the constituent quark model with and without chiral interactions ($\pi-$ and $\sigma-$ exchange) between quarks. The aim of this study was to look for a possible dibaryon with these quantum numbers. It was argued [11], that such a dibaryon is seen as a narrow peak in double charge exchange reactions (π^+, π^-) on various nuclei.

We have classified all possible six–quark states $J^P=0^-$, T=0 with N=1 and N=3 harmonic oscillator excitations using different reduction chains. We then have diagonalized the microscopic Hamiltonian including the linear quark–quark confinement, chiral interactions between quarks, and the effective one–gluon exchange–potential in the basis consisting of the lowest (N=1) state and the 10 presumably most important configurations from the N=3 shell. We have argued, that the calculated mass–eigenvalue should be compared to the 2–baryon N(939)+N*(1535) threshold.

We have found, that if one fits all parameters to describe the nucleon and its lowest resonances, the mass of such a "dibaryon" lies above the N(939)+N*(1535) threshold, when taking our restricted basis of 11 states. Since we gain around 100 – 150 MeV by admixing the 10 chosen states, it seems not to be unrealistic, that a more complete basis would lead to a "dibaryon mass" below the N(939)N*(1535) threshold of 2474 MeV. So, our calculations do not deny the existence of a dibaryon

24

with $J^P=0^-$, T=0. Its mass could be some 10 MeV's below the NN* threshold (within the framework of our model). But the discrepancy with the "experimentally found" mass is still large.

We cannot claim for sure, that the constituent quark model "excludes" the interpretation of the peak in (π^+, π^-) reactions as a signal from the dibaryon with the above mentionned quantum numbers, since our basis is not complete, and as a consequence, we lose probably some of the possible short–range and few–quark correlations in the six–quark system. Another point to mention is the following: The 6q–system differs qualitatively from the description of a baryon in the sense, that the 6q–color symmetry is a mixed symmetry, in contrast to the fully antisymmetric color–wavefunction of a baryon. Probably, the confinement for a 6q–system (in addition to nuclear medium effects in the nucleus) differs qualitatively from the confinement mechanism in the baryons. At this point, we should add, that the choice of a quadratic or a so–called color–screened exponential [33] rather than a linear confinement potential does not change qualitatively our results. If the assumption of a dibaryon resonance at 2065 MeV is confirmed, we could probably improve our understanding of confinement in multi–quark systems.

We have also analyzed a possible $J^P=0^-$, T=2 dibaryon. The corresponding lowest shell model state lies below the $J^P=0^-$, T=0 one, provided that the one–pion–exchange forces play an essential role in the 6q–system.

Finally, we have shown, that we could describe an "observed dibaryon" with a mass close to $M_{d'}$= 2065 MeV, by reducing the confinement strength a_c (string tension), down to $a_c \simeq 25$ MeV/fm [15] (see parameter set V). But, such a confinement strength does not give the correct energy for the negative parity one–quantum excitation N*(1535). Thus, we observe two opposing tendencies within our model: We need a small confinement strength (and therefore a large size parameter b_N for the nucleon) to get a rather large dibaryon, which is consequently rather light in mass. On the other hand, we need a rather small nucleon size b_N to describe the baryon spectrum, especially the N* excited states. This result suggests, that this six–quark

system is strongly influenced by the confinement mechanism. The above discrepancy can not be eliminated by another choice of radial dependence of the confinement potential (quadratic or "color–screened" exponential) alone. If the d' dibaryon is confirmed, we suggest that the color–structure $\lambda_i^a \cdot \lambda_j^a$ of the confinement potential should be modified in multi–quark systems like the d'. We should stress here once more, that it seems impossible to describe the nucleon resonances (the 3q–sector) and this dibaryon resonance within the same model with the same parameters.

Appendix A

In this appendix, we give the mass formulae for the baryon systems

$$N(939), \Delta(1232), \text{ and } N^*(1535), N^{**}(1440) \tag{A.1}$$

within the constituent quark model, as described in the parameter fitting section.

The following harmonic oscillator basis states

$$
\begin{aligned}
\mid N_0 > \; &= \; \mid N = 0, (\lambda\mu) = (00), [3]_X, L = 0, S = \frac{1}{2}, T = \frac{1}{2} \, [3]_{ST}\rangle \,, \\
\mid \Delta_0 > \; &= \; \mid N = 0, (\lambda\mu) = (00), [3]_X, L = 0, S = \frac{3}{2}, T = \frac{3}{2} \, [3]_{ST}\rangle \,, \\
\mid N_1^{(-)} > \; &= \; \mid N = 1, (\lambda\mu) = (10), [21]_X, L = 1, S = \frac{1}{2}, T = \frac{1}{2} \, [21]_{ST}\rangle \,, \\
\mid N_2^{(-)} > \; &= \; \mid N = 1, (\lambda\mu) = (10), [21]_X, L = 1, S = \frac{3}{2}, T = \frac{1}{2} \, [21]_{ST}\rangle \,, \\
\mid N_1 > \; &= \; \mid N = 2, (\lambda\mu) = (20), [3]_X, L = 0, S = \frac{1}{2}, T = \frac{1}{2} \, [3]_{ST}\rangle \,, \\
\mid N_2 > \; &= \; \mid N = 2, (\lambda\mu) = (20), [21]_X, L = 0, S = \frac{1}{2}, T = \frac{1}{2} \, [21]_{ST}\rangle \,, \tag{A.2}
\end{aligned}
$$

are needed for the calculation of the appropriate matrix elements which determine the masses of the baryons N(939), Δ(1232), N*(1535) and N**(1440) in our chosen parameter fitting scheme. The ground states are denoted by the subscript "0". The detailed wave functions can be found in the second paper of [3].

Our Ansatzes for the different baryons are:

$$|N\rangle = |N_0\rangle \quad : \quad |\Delta\rangle = |\Delta_0\rangle$$

$$|N^*\rangle = \lambda\,|N_1^{(-)}\rangle + \mu\,|N_2^{(-)}\rangle$$

$$|N^{**}\rangle = \alpha\,|N_1\rangle + \beta\,|N_2\rangle \tag{A.3}$$

To be rather short, we give here all necessary (diagonal and off–diagonal) matrix elements in their analytical form in one table.

$$
\begin{aligned}
h_{ij}^{N,\Delta,N^{(-)},N^{**}} =\ & 3m_q\,\delta_{ij} + k_{ij}\,\frac{1}{m_q b^2} + a_{ij}\,\frac{a_c b}{\sqrt{2\pi}} - 2C\,\delta_{ij} + b_{ij}\,\frac{\alpha_s}{\sqrt{2\pi}\,b} + \\
& + (ce_{ij} + cm_{ij})\frac{\alpha_s}{\sqrt{2\pi}m_q^2 b^3} + V_C^\pi + V_T^\pi + V_C^\sigma
\end{aligned}
\tag{A.4}
$$

The notations for the pion central–contribution are the same as in (B.9) and for the σ–exchange the notations are the same as in (B.10). As in appendix B, we do not give the lengthy formulae for the pion–tensor contribution in explicit form.

		k	a	b	ce	cm	p,p1,p3,p5	s,s1,s3,s5
N	h_{00}^N	$\frac{3}{2}$	8	–4	1	$\frac{-2}{3}$	5,1,0,0	–3,1,0,0
Δ	h_{00}^Δ	$\frac{3}{2}$	8	–4	1	$\frac{+2}{3}$	1,1,0,0	–3,1,0,0
	$h_{11}^{N(-)}$	2	$\frac{28}{3}$	$\frac{-10}{3}$	$\frac{1}{2}$	$\frac{-1}{3}$	$\frac{1}{2},5,-1,0$	$\frac{-3}{2},1,\frac{1}{3},0$
N*	$h_{22}^{N(-)}$	2	$\frac{28}{3}$	$\frac{-10}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2},1,-1,0$	$\frac{-3}{2},1,\frac{1}{3},0$
	$h_{12}^{N(-)}$	0	$\frac{2\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	$\frac{-1}{2\sqrt{2}}$	$\frac{-1}{3\sqrt{2}}$	$\frac{-1}{2\sqrt{2}},1,1,0$	$\frac{3}{2\sqrt{2}},1,\frac{-1}{3},0$
	h_{11}^N	$\frac{5}{2}$	10	$\frac{-11}{3}$	$\frac{5}{4}$	$\frac{-5}{6}$	$\frac{5}{2},\frac{5}{2},-1,\frac{1}{6}$	$\frac{-3}{2},\frac{5}{2},-1,\frac{1}{6}$
N**	h_{22}^N	$\frac{5}{2}$	$\frac{31}{3}$	$\frac{-19}{6}$	$\frac{5}{8}$	$\frac{-5}{12}$	$\frac{1}{4},\frac{25}{2},-7,\frac{5}{6}$	$\frac{-3}{4},\frac{5}{2},\frac{-1}{3},\frac{1}{6}$
	h_{12}^N	0	0	0	0	$\frac{1}{3\sqrt{2}}$	$-\sqrt{2},\frac{1}{2},-1,\frac{1}{6}$	0

Appendix B

In this appendix, we present all matrix elements

$$
\begin{aligned}
H_{ji} \equiv H_{ij} =\ & \langle i\,|\,H\,|\,j\rangle \quad ; \quad i,j = 1\ldots 11 \\
=\ & 6m_q\,\delta_{ij} + (E^{kin} + V^{conf} + V^{OGEP} + V^\pi + V^\sigma)_{ij} \tag{B.1}
\end{aligned}
$$

for the different parts of the Hamiltonian eq.(16). in the tables B.1 through B.4, excluding the pion tensor contribution again. As mentioned above, the labelling of the basis states, as they are used here, is given in table 4.

The kinetic energy contribution is given by

$$
E_{ij}^{kin} = \begin{cases} \frac{17}{4} \frac{1}{m_q b^2} & \text{if } i = j = 1 \\ \frac{21}{4} \frac{1}{m_q b^2} & \text{if } i = j = 2 \dots 11 \\ 0 & \text{if } i \neq j \end{cases} \tag{B.2}
$$

The coefficient a_{ij} of the confinement contribution

$$
V_{ij}^{conf} = \langle i | \sum_{k<l}^{6} -\frac{\lambda_k^a \cdot \lambda_l^a}{4} (a_c r - C) | j \rangle = a_{ij} \frac{a_c b}{\sqrt{2\pi}} - 4C \, \delta_{ij} \tag{B.3}
$$

is given by

Tables B.2a – B.2d give the different parts of the one–gluon exchange potential, starting with the color Coulomb contribution

$$
V_{ij}^{CC} = \langle i | \sum_{k<l}^{6} \frac{\alpha_s}{4} \lambda_k^a \cdot \lambda_l^a \frac{1}{r} | j \rangle = -b_{ij} \frac{\alpha_s}{\sqrt{2\pi} b} \tag{B.4}
$$

The next table gives the so–called color–electric part

$$
V_{ij}^{CE} = \langle i | \sum_{k<l}^{6} -\frac{\alpha_s}{4} \frac{\pi}{m_q^2} \lambda_k^a \cdot \lambda_l^a \delta(\vec{r}) | j \rangle = ce_{ij} \frac{\alpha_s}{\sqrt{2\pi} m_q^2 b^3} \tag{B.5}
$$

This is the color–magnetic contribution

$$
V_{ij}^{CM} = \langle i | \sum_{k<l}^{6} -\frac{\alpha_s}{6} \frac{\pi}{m_q^2} \lambda_k^a \cdot \lambda_l^a \boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_l \delta(\vec{r}) | j \rangle = cm_{ij} \frac{\alpha_s}{\sqrt{2\pi} m_q^2 b^3} \tag{B.6}
$$

The next table gives the gluon tensor contributions

$$
V_{ij}^{GT} = \langle i | \sum_{k<l}^{6} -\frac{\alpha_s}{16 m_q^2} \lambda_k^a \cdot \lambda_l^a \hat{S}_{kl} \frac{1}{r^3} | j \rangle = ct_{ij} \frac{\alpha_s}{\sqrt{2\pi} m_q^2 b^3} \tag{B.7}
$$

Introducing the following notations

$$
\begin{aligned}
I_{(n)}(m_\pi, b_N) &= \int_0^\infty r^n \exp(-\frac{r^2}{2b_N^2} - m_\pi r) dr \\
I_{(0)}(m_\pi, b_N) &= \sqrt{\frac{\pi}{2}} \, b_N \, \text{erfc}\left(\frac{m_\pi b_N}{\sqrt{2}}\right) \exp(\frac{m_\pi^2 b_N^2}{2}) \\
\text{erfc}(z) &= \frac{2}{\sqrt{\pi}} \int_z^\infty \exp(-x^2) dx \\
I_{(n+1)} &= -\frac{\partial}{\partial m_\pi} I_{(n)}
\end{aligned} \tag{B.8}
$$

for the radial integrals needed in the calculation of the Yukawa–like pion– and sigma–potential matrix elements, allows us to present the chiral contributions in the following form: First, for the pion central contribution, we write

$$V_{ij}^\pi = \frac{\Lambda^2}{\Lambda^2 - m_\pi^2} \frac{g^2}{4\pi} \frac{p_{ij}}{4m_q^2 b^3} \sqrt{\frac{2}{\pi}} \left\{ m_\pi^2 \left(p1_{ij} I_{(1)} + p3_{ij} \frac{I_{(3)}}{b^2} + p5_{ij} \frac{I_{(5)}}{b^4} \right) - \Lambda^2 \left(m_\pi \leftrightarrow \Lambda \right) \right\}$$
(B.9)

So, in the four next tables (table B.3a – B.3d), we give the coefficients $p, p1, p3, p5$ respectively. Let us mention once more, that since the pion tensor contributions involve all above integrals from $I_{(1)}$ up to $I_{(5)}$, and gives rather negligible contributions, we do not give the analytical expressions here.

Finally, table B.4 concerns the σ–potential, where we give only the non–zero matrix elements. The notations are the same as for the pion, i.e.

$$V_{ij}^\sigma = -\frac{\Lambda^2}{\Lambda^2 - m_\sigma^2} \frac{g^2}{4\pi} \frac{s_{ij}}{b^3} \sqrt{\frac{2}{\pi}} \left\{ \left(s1_{ij} I_{(1)} + s3_{ij} \frac{I_{(3)}}{b^2} + s5_{ij} \frac{I_{(5)}}{b^4} \right) - \left(m_\sigma \leftrightarrow \Lambda \right) \right\}$$ (B.10)

Appendix C

In this appendix, we present the expectation values of the Hamiltonian (16) for the two states of eq.(42) and eq.(43):

$$
\begin{aligned}
M_1^{T=2} &= \langle 1|H^{(6q)}|1\rangle \\
&= 6m_q + \frac{17}{4m_q b^2} + \frac{83 a_c b}{5\sqrt{2\pi}} - 4C - \frac{77\alpha_s}{10\sqrt{2\pi} b} + \frac{1321\alpha_s}{360\sqrt{2\pi} m_q^2 b^3} \\
&\quad + V_C^\pi(1) + V^\sigma(1) + \frac{23\alpha_s}{45\sqrt{2\pi} m_q^2 b^3} + V_T^\pi(1)
\end{aligned}
$$
(C.1)

$$
\begin{aligned}
M_2^{T=2} &= \langle 2|H^{(6q)}|2\rangle \\
&= 6m_q + \frac{17}{4m_q b^2} + \frac{256 a_c b}{15\sqrt{2\pi}} - 4C - \frac{112\alpha_s}{15\sqrt{2\pi} b} + \frac{148\alpha_s}{45\sqrt{2\pi} m_q^2 b^3} \\
&\quad + V_C^\pi(2) + V^\sigma(2) - \frac{\alpha_s}{15\sqrt{2\pi} m_q^2 b^3} + V_T^\pi(2)
\end{aligned}
$$
(C.2)

One recognizes the CC–term $\simeq \frac{\alpha_s}{b}$ and the added CE and CM–term $\simeq \frac{\alpha_s}{b^3}$. The second $\frac{\alpha_s}{b^3}$–term comes from the tensor part of the one–gluon exchange potential.

29

With the notations of appendix B, eq.(B.9) and eq.(B.10), the central parts of the chiral interactions are given in the following list

	p	p1	p3	p5	s	s1	s3	s5
$V_C^\pi(1)$	$\frac{1}{15}$	13	$\frac{-14}{3}$	0				
$V^\sigma(1)$					3	4	$\frac{1}{3}$	0
$V_C^\pi(2)$	$\frac{-1}{15}$	8	7	0				
$V^\sigma(2)$					3	4	$\frac{1}{3}$	0

The tensor contributions of the pion are given by

$$V_T^\pi(i) = \mathrm{pt}_{(i)}\, \frac{\Lambda^2}{\Lambda^2 - m_\pi^2}\, \frac{g^2}{4\pi}\, \frac{1}{4m_q^2 b^5}\sqrt{\frac{2}{\pi}}\left\{ m_\pi^2\left(I_{(3)} + \frac{3I_{(2)}}{m_\pi} + \frac{3I_{(1)}}{m_\pi^2}\right) - \Lambda^2\left(m_\pi \leftrightarrow \Lambda\right)\right\}$$

(C.3)

with

$$\mathrm{pt}_{(1)} = \frac{-58}{45} \qquad , \qquad \mathrm{pt}_{(2)} = \frac{1}{15}$$

References

[1] S.Godfrey, N.Isgur, *Phys.Rev.***D32** (1985), 189;
S.Capstick, N.Isgur, *Phys.Rev.***D34** (1986), 2809;
W.Lucha, F.F.Schöberl, D.Gromes, *Phys.Rep.***200** (1991), 127;
J.–M.Richard, *Phys.Rep.***212** (1992), 1

[2] E.Witten, *Nucl.Phys.***B223** (1983), 422, 433;
I.Zahed, G.E.Brown, *Phys.Rep.***142** (1986), 1

[3] L.Ya.Glozman, V.G.Neudatchin, I.T.Obukhovsky, *Phys.Rev.***C48** (1993) 389;
L.Ya.Glozman and E.I.Kuchina, *Phys.Rev.***C49** (1994) 1149;
L.Ya.Glozman and A.Faessler, *Phys.Lett.***B** (1995), in print.

[4] R.L.Jaffe *Phys.Rev.Lett.***38** (1977), 195

[5] P.J.G.Mulders et al *Phys.Rev.Lett.***40** (1978),1543; *Phys.Rev.***D21** (1980), 2653

[6] Yu.S.Kalashnikova, I.M.Narodetskii and Yu.A.Simonov *Sov.J.Nucl.Phys.***46** (1987), 689

[7] P.Gonzales, P.LaFrance and E.Lomon *Phys.Rev.* **D35** (1987), 2142;
E.Lomon *J.Physique* **51** (1990), Supple. Colloque C6 363

[8] L.A.Kondratyuk, B.V.Martemyanov and M.G.Schepkin, *Sov.J.Nucl.Phys.***45** (1987), 776

[9] S.Takeuchi and M.Oka, *Phys.Rev.Lett.***66** (1991), 1271

[10] U.Straub, Z.–Ye Zhang, K.Bräuer, A.Faessler, S.B.Khadkikar, G.Lübeck, *Nucl.Phys***A508** (1990), 385c

[11] R.Bilger, H.A.Clement and M.G.Schepkin *Phys.Rev.Lett.***71** (1993), 42;
R.Bilger et al *Z.Phys.***A343** (1992), 491

[12] M.Oka and K.Yazaki *Progr.Theor.Phys.***66** (1981), 556;

A.Faessler, F.Fernandez, G.Lübeck, K.Shimizu, *Phys.Lett.***B112** (1982), 201;

C.S.Warke, R.Shanker, *Phys.Rev.***C21** (1980), 2643;

J.E.T.Ribeiro, *Z.Phys.***C5** (1980), 27;

Z.–Ye Zhang, A.Faessler, U.Straub, L.Ya.Glozman, *Nucl.Phys.***A578** (1994), 573;

A.Valcarce, A.Buchmann, F.Fernandez and Amand Faessler, *Phys.Rev.***C50** (1994), 2246

[13] A.Faessler, F.Fernandez, G.Lübeck, K.Shimizu, *Nucl.Phys.***A402**(1983),555;

A.Faessler, F.Fernandez, *Phys.Lett.***124B**(1983),145

[14] I.T.Obukhovsky and A.M.Kusainov, *Sov.J.Nucl.Phys.* **47** (1988), 313 ;

A.M.Kusainov, V.G.Neudatchin, I.T.Obukhovsky, *Phys.Rev.***C44** (1991),2343

[15] A.Buchmann, E.Hernández and K.Yazaki, *Phys.Lett.***B269** (1991), 35;

A.Buchmann, E.Hernández and K.Yazaki, *Nucl.Phys.***A569** (1994), 661;

A.Buchmann, *Czech.J.Phys.***44** (1994), 347

[16] O.W.Greenberg, H.J.Lipkin, *Nucl.Phys.***A370** (1981), 349

[17] L.Ya.Glozman, A.Buchmann and A.Faessler, *Jour.Phys.***G20**(1994),L49

[18] A.Manohar and H.Georgi, *Nucl.Phys.***B234** (1984), 189

[19] D.I.Dyakonov, V.Yu.Petrov, *Nucl.Phys.***B272** (1986), 457

[20] E.V.Shuryak, *Phys.Rep.***115** (1984), 151

[21] U.Vogl, W.Weise, *Prog.Part.Nuc.Phys.***27** (1991), 195

[22] M.Gell–Mann, M.Levy, *Nuovo Cim.***16** (1960), 53

[23] A.deRujula, H.Georgi and S.L.Glashow, *Phys.Rev.***D12** (1975), 147

[24] A.Valcarce, A.J.Buchmann, F.Fernandez, A.Faessler, (1994), submitted to *Phys.Rev.***C**

[25] L.Ya.Glozman, D.O.Riska, (1994), preprint HU-TFT-94-47 [LANL hep-ph 9411279]; preprint HU-TFT-94-48 [LANL hep-ph 9412231]

[26] Z.–Ye Zhang, K.Bräuer, A.Faessler, K.Shimizu, *Nucl.Phys.***A443**(1985),557

[27] I.V.Kurdyumov, Yu.F.Smirnov, K.V.Shitikova and S.El.Samarai, *Nucl.Phys.* **A145** (1970), 593;
V.G.Neudatchin, Yu.F.Smirnov, N.F.Golovanova *Adv.Nucl.Phys.***11** (1979),1;
O.F.Nemets, V.G.Neudatchin, A.T.Rudchik, Yu.F.Smirnov, Yu.M.Tchuvilsky, Nucleon Clustering in Light Nuclei and Multi–nucleon Transfer Reactions (in russian), Nankova Dumka, Kiev, 1988.

[28] I.T.Obukhovsky, Yu.F.Smirnov and Yu.M.Tchuvilsky, *J.Phys.***A15** (1982), 7

[29] Yu.F.Smirnov Yu.I.Kharitonov, *Leningrad Inst.Nucl.Phys.*(1986), preprint 1194

[30] M.Harvey *Nucl.Phys.***A352** (1981), 301; 326

[31] I.T.Obukhovsky *Z.Phys.***A308** (1982), 253

[32] H.Garcilazo and L.Mathelitsch, *Phys.Rev.Lett.***72** (1994), 2971

[33] Z.–Ye Zhang *et al*, *Nucl.Phys.***A561** (1993), 595

	b_N	Λ	m_q	α_s	a_c	C	$N^*(1535) = \lambda N_1^{(-)} + \mu N_2^{(-)}$				N^{**}
Set	[fm]	[fm^{-1}]	[MeV]		[MeV/fm]	[MeV]	λ	μ	$N^*(1535)$	$N^*(1650)$	[MeV]
I	.45	5.07	338	.127	462	549	.98	.22	1535	1697	1678
II	.47	7.6	296	.074	423	500	-.96	.30	1535	1713	1660
III	.5	3.55	274	.334	654	676	.998	.06	1535	1683	1797
IV	.5	/	230	.474	1223	1247	-.86	.51	1535	1844	2120
V	.6	10.14	313	.816	25	-32	.98	.22	1275	1436	1292

Table 1: Different sets of parameters for the quark–quark interactions of eqs.(10)–(15), fitted as described in the text. The first 6 columns show the harmonic oscillator length b_N used to describe the baryons, the cutoff Λ parametrizing the finite size of the pion–quark and sigma–quark vertex, the constituent quark mass m_q, the strong coupling constant α_s of the one–gluon exchange and the strength (slope) a_c and offset C of the confinement potential, respectively. The diagonalisation of the mass matrix in the basis of two states $N_1^{(-)}$ and $N_2^{(-)}$ gives the energies of the two negative parity resonances N*(1535) and N*(1650). The next four columns give the amplitudes and masses of these two resonances, respectively. The last column gives for illustration the mass of the lowest two–quantum excitation (N=2) for the parameters given, calculated in a basis of the two most important states.

$[f]_X$	[51]	[51]	[42]	[42]	[42]	[411]	[411]	[321]	[321]	[3111]
$(\lambda\mu)$	(11), (30)	(11)	(11), (30)	(11)	(30)	(11), (30)	(11)	(11)	(11)	(00)
LST	110	220	110	220	330	110	220	110	220	000
$[f]_{CS}$	[321]	[321], [2²1²]	[42], [321], [31³], [2³], [21⁴]	[321]	[2³]	[321], [31³]	[321], [2²1²]	[42], [321]₁, [321]₂, [31³], [21⁴]	[321]₁, [321]₂, [2²1²]	[41²]
$[f]_{ST}$	[321]	[42], [321]	[51], [411], [33], [321], [2²1²]	[321]	[33]	[411], [321]	[42], [321]	[51], [411], [321]₁, [321]₂, [2²1²]	[42], [321]₁, [321]₂	[31³]
$[f]_{XS}$	[42]	[6], [42]	[6], [42]₁, [42]₂, [31³], [222]	[42]	[6]	[42], [31³]	[42], [31³]	[42]₁, [42]₂, [31³]₁, [31³]₂, [222]	[42], [31³], [222]	[31³]
states	2	2	10	1	1	4	2	5	3	1

Table 2: Here, we give a complete list of all possible six–quark states with three internal harmonic oscillator quanta N=3 and the quantum numbers $J^P=0^-$, T=0. The first line defines the spatial permutational symmetry $[f]_X$, and the second the corresponding possible Elliot symbols $(\lambda\mu)$ for the harmonic oscillator. The third line shows the total orbital angular momentum L, the total spin S and the total isospin T. The next three lines give the intermediate permutational symmetry needed for an unambigious classification of the states within a certain reduction scheme. Line 4 corresponds to the reduction chain (29) with explicit intermediate color–spin symmetry, line 5 to (30) with intermediate spin–isospin symmetry and line 6 to (31) with intermediate spin–orbital symmetry. The reduction chain (31) was used in the present calculation. The last line gives the number of states in each column, respectively.

	excess energies		groundstate		T=2		mixing results	
	δM $(b_6=b_N)$	δM_{s^6} $(b_6=b_N)$	b_6	$M_{N=1}^{T=0}$	$M_1^{T=2}$	$M_2^{T=2}$	b_6	M
Set	$[MeV]$	$[MeV]$	$[fm]$	$[MeV]$	$[MeV]$	$[MeV]$	$[fm]$	$[MeV]$
I	525	509	.59	2705	2564	2744	.60	2588
II	566	555	.65	2680	2530	2705	.65	2565
III	624	603	.60	2898	2812	2949	.62	2760
IV	850	924	.56	3288	3303	3333	.59	3114
V	441	400	1.24	2162	2146	2169	1.24	2132

Table 3: For different sets of parameters I to V defined in table 1, the first column δM gives the mass difference of the calculated lowest $J^P=0^-$, T=0 state and the N(939)N*(1535) threshold. For comparison, the second column δM_{s^6} gives the mass excess for six quarks, coupled to deuteron–like quantum numbers, in one harmonic oscillator s^6 with respect to the N(939)N(939) threshold at 1878 MeV. Both mass excesses in columns 1 and 2 are calculated for the oscillator length b_6 equal to the value b_N minimizing the nucleon mass. Columns 3 and 4 show the oscillator length b_6 obtained by minimizing the single N=1, $J^P=0^-$, T=0, $[51]_X$, $(\lambda\mu)=(10)$, $[f']_{XS}=[42]$ configuration and the corresponding mass. Columns 5 and 6 list the masses of the two N=1, $J^P=0^-$, T=2 states. Here again, the oscillator length (not given) is varied to minimize both energies separately. Columns 7 and 8 give the final result for the oscillator length b_6, minimizing the lowest $J^P=0^-$, T=0 energy eigenvalue, and the corresponding mass M. This mass should be compared with the observed experimental resonance energy of 2065 MeV.

$|\text{harm.osc.}_i\rangle$	$i=1$	2	3	4	5	6	7	8	9	10	11
N	1	3	3	3	3	3	3	3	3	3	3
$[f]_X$	[51]	[42]	[42]	[42]	[42]	[42]	[42]	[42]	[42]	[42]	[42]
$(\lambda\mu)$	(10)	(30)	(11)	(30)	(11)	(30)	(11)	(30)	(11)	(30)	(11)
L, S, T						1, 1, 0					
$[f']_{XS}$	[42]	[6]	[6]	$[2^3]$	$[2^3]$	$[31^3]$	$[31^3]$	$[42]_1$	$[42]_1$	$[42]_2$	$[42]_2$

Table 4: Quantum numbers and our labelling of the states included in the configuration mixing to calculate the lowest $J^P=0^-$, T=0 six–quark configuration. The first line gives our labelling of the states. The next line lists the respective number of excited harmonic oscillator quanta N, and line three represents the orbital permutational symmetry of the Young tableaux. The fourth line shows the Elliot quantum numbers of the harmonic oscillator $(\lambda\mu)$, while line five gives the total orbital angular momentum L, the total spin S and the total isospin T of the configurations. The last line denotes the intermediate symmetries of the Young tableaux $[f']_{XS}$ in the orbital–spin space.

a_{ij}	$j=1$	2	3	4	5	6	7	8	9	10	11
$i=1$	$\frac{53}{3}$	$\frac{5}{3\sqrt{15}}$	$\frac{-16}{3\sqrt{30}}$	$\frac{5}{3\sqrt{15}}$	$\frac{-16}{3\sqrt{30}}$	$\frac{5}{3\sqrt{30}}$	$\frac{-8}{3\sqrt{15}}$	$\frac{5}{6\sqrt{15}}$	$\frac{-8}{3\sqrt{30}}$	$\frac{5}{12\sqrt{15}}$	$\frac{-3}{2\sqrt{30}}$
2		$\frac{12784}{675}$	$\frac{4\sqrt{2}}{135}$	0	0	0	0	$\frac{-343}{450}$	$\frac{-2\sqrt{2}}{45}$	$\frac{253}{300}$	0
3			$\frac{512}{27}$	0	0	0	0	$\frac{-2\sqrt{2}}{45}$	$\frac{2}{9}$	0	$\frac{1}{3}$
4				$\frac{39917}{2160}$	$\frac{23}{540\sqrt{2}}$	$\frac{-9}{40\sqrt{2}}$	$\frac{-1}{20}$	$\frac{121}{144}$	$\frac{11}{180\sqrt{2}}$	$\frac{-3}{40}$	$\frac{-1}{30\sqrt{2}}$
5					$\frac{2039}{108}$	$\frac{-1}{20}$	$\frac{1}{2\sqrt{2}}$	$\frac{11}{180\sqrt{2}}$	$\frac{-1}{36}$	$\frac{-1}{30\sqrt{2}}$	$\frac{1}{6}$
6						$\frac{20947}{1080}$	$\frac{37}{270\sqrt{2}}$	$\frac{-3}{40\sqrt{2}}$	$\frac{-1}{60}$	$\frac{25}{18\sqrt{2}}$	$\frac{-1}{90}$
7							$\frac{1009}{54}$	$\frac{-1}{60}$	$\frac{1}{6\sqrt{2}}$	$\frac{-1}{90}$	$\frac{-2\sqrt{2}}{9}$
8								$\frac{203701}{10800}$	$\frac{47}{540\sqrt{2}}$	$\frac{-26}{75}$	$\frac{1}{30\sqrt{2}}$
9									$\frac{2027}{108}$	$\frac{1}{30\sqrt{2}}$	$\frac{-1}{3}$
10										$\frac{208441}{10800}$	$\frac{17}{135\sqrt{2}}$
11											$\frac{2021}{108}$

Table B.1: Analytic expressions for the confinement matrix elements

b_{ij}	$j=1$	2	3	4	5	6	7	8	9	10	11
$i=1$	$\frac{43}{6}$	$\frac{-5}{6\sqrt{15}}$	$\frac{4}{3}\sqrt{\frac{2}{15}}$	$\frac{-5}{6\sqrt{15}}$	$\frac{4}{3}\sqrt{\frac{2}{15}}$	$\frac{-5}{12}\sqrt{\frac{2}{15}}$	$\frac{4}{3\sqrt{15}}$	$\frac{-5}{12\sqrt{15}}$	$\frac{2}{3}\sqrt{\frac{2}{15}}$	$\frac{-5}{24\sqrt{15}}$	$\frac{3}{8}\sqrt{\frac{2}{15}}$
2		$\frac{1528}{225}$	$\frac{2\sqrt{2}}{45}$	0	0	0	0	$\frac{21}{100}$	$\frac{\sqrt{2}}{15}$	$\frac{-319}{600}$	0
3			$\frac{304}{45}$	0	0	0	0	$\frac{\sqrt{2}}{15}$	$\frac{-1}{15}$	0	$\frac{11}{30}$
4				$\frac{10123}{1440}$	$\frac{23}{360\sqrt{2}}$	$\frac{-7}{80\sqrt{2}}$	$\frac{3}{40}$	$\frac{-11}{32}$	$\frac{-11}{120\sqrt{2}}$	$\frac{-13}{240}$	$\frac{1}{20\sqrt{2}}$
5					$\frac{2477}{360}$	$\frac{3}{40}$	$\frac{-7}{20\sqrt{2}}$	$\frac{-11}{120\sqrt{2}}$	$\frac{-11}{120}$	$\frac{1}{20\sqrt{2}}$	$\frac{-7}{60}$
6						$\frac{4853}{720}$	$\frac{37}{180\sqrt{2}}$	$\frac{-13}{240\sqrt{2}}$	$\frac{1}{40}$	$\frac{51}{60\sqrt{2}}$	$\frac{1}{60}$
7							$\frac{1243}{180}$	$\frac{1}{40}$	$\frac{-7}{60\sqrt{2}}$	$\frac{1}{60}$	$\frac{22\sqrt{2}}{135}$
8								$\frac{49859}{7200}$	$\frac{47}{360\sqrt{2}}$	$\frac{8}{25}$	$\frac{-1}{20\sqrt{2}}$
9									$\frac{2489}{360}$	$\frac{-1}{20\sqrt{2}}$	$\frac{59}{90}$
10										$\frac{48719}{7200}$	$\frac{17}{90\sqrt{2}}$
11											$\frac{2483}{360}$

Table B.2a: Analytic expressions for the color coulomb matrix elements

ce_{ij}	$j=1$	2	3	4	5	6	7	8	9	10	11
$i=1$	$\frac{99}{72}$	$\frac{-5}{8\sqrt{15}}$	$\frac{2}{\sqrt{30}}$	$\frac{-5}{8\sqrt{15}}$	$\frac{2}{\sqrt{30}}$	$\frac{-5}{8\sqrt{30}}$	$\frac{1}{\sqrt{15}}$	$\frac{-5}{16\sqrt{15}}$	$\frac{1}{\sqrt{30}}$	$\frac{-5}{32\sqrt{15}}$	$\frac{9}{16\sqrt{30}}$
2		$\frac{58}{45}$	$\frac{-1}{9\sqrt{2}}$	0	0	0	0	$\frac{7}{240}$	$\frac{\sqrt{2}}{12}$	$\frac{-77}{160}$	0
3			$\frac{56}{45}$	0	0	0	0	$\frac{\sqrt{2}}{12}$	$\frac{-1}{60}$	0	$\frac{-17}{40}$
4				$\frac{1699}{1152}$	$\frac{-23}{288\sqrt{2}}$	$\frac{-21}{64\sqrt{2}}$	$\frac{3}{32}$	$\frac{-77}{384}$	$\frac{-11}{96\sqrt{2}}$	$\frac{-7}{64}$	$\frac{1}{16\sqrt{2}}$
5					$\frac{2017}{1440}$	$\frac{3}{32}$	$\frac{-27}{80\sqrt{2}}$	$\frac{-11}{96\sqrt{2}}$	$\frac{-71}{480}$	$\frac{1}{16\sqrt{2}}$	$\frac{-9}{80}$
6						$\frac{797}{576}$	$\frac{-37}{144\sqrt{2}}$	$\frac{-7}{64\sqrt{2}}$	$\frac{1}{32}$	$\frac{35}{48\sqrt{2}}$	$\frac{1}{48}$
7							$\frac{983}{720}$	$\frac{1}{32}$	$\frac{-9}{80\sqrt{2}}$	$\frac{1}{48}$	$\frac{19}{30\sqrt{2}}$
8								$\frac{8411}{5760}$	$\frac{-47}{288\sqrt{2}}$	$\frac{-3}{16}$	$\frac{-1}{16\sqrt{2}}$
9									$\frac{2029}{1440}$	$\frac{-1}{16\sqrt{2}}$	$\frac{13}{40}$
10										$\frac{7991}{5760}$	$\frac{-17}{72\sqrt{2}}$
11											$\frac{1963}{1440}$

Table B.2b: Analytic expressions for the color electric matrix elements (δ–force)

cm_{ij}	$j=1$	2	3	4	5	6	7	8	9	10	11
$i=1$	$\frac{5}{36}$	$\frac{-5}{12\sqrt{15}}$	$\frac{4}{3\sqrt{30}}$	$\frac{-5}{12\sqrt{15}}$	$\frac{4}{3\sqrt{30}}$	$\frac{5}{4\sqrt{30}}$	$\frac{-2}{\sqrt{15}}$	$\frac{-25}{24\sqrt{15}}$	$\frac{4}{3\sqrt{30}}$	$\frac{85}{144\sqrt{15}}$	$\frac{-53}{72\sqrt{30}}$
2		$\frac{116}{135}$	$\frac{-2}{27\sqrt{2}}$	0	0	0	0	$\frac{7}{360}$	$\frac{\sqrt{2}}{18}$	$\frac{-77}{240}$	0
3			$\frac{112}{135}$	0	0	0	0	$\frac{\sqrt{2}}{18}$	$\frac{-1}{90}$	0	$\frac{-17}{60}$
4				$\frac{-1807}{1728}$	$\frac{49}{432\sqrt{2}}$	$\frac{21}{32\sqrt{2}}$	$\frac{-3}{16}$	$\frac{-77}{576}$	$\frac{-11}{144\sqrt{2}}$	$\frac{-7}{96}$	$\frac{1}{24\sqrt{2}}$
5					$\frac{-142}{4320}$	$\frac{-3}{16}$	$\frac{27}{40\sqrt{2}}$	$\frac{-11}{144\sqrt{2}}$	$\frac{-71}{720}$	$\frac{1}{24\sqrt{2}}$	$\frac{-9}{40}$
6						$\frac{-13}{288}$	$\frac{-11}{72\sqrt{2}}$	$\frac{7}{32\sqrt{2}}$	$\frac{-1}{16}$	$\frac{-35}{24\sqrt{2}}$	$\frac{-1}{24}$
7							$\frac{723}{720}$	$\frac{-1}{16}$	$\frac{9}{40\sqrt{2}}$	$\frac{-1}{24}$	$\frac{-19}{15\sqrt{2}}$
8								$\frac{3731}{8640}$	$\frac{-23}{432\sqrt{2}}$	$\frac{109}{144}$	$\frac{-7}{72\sqrt{2}}$
9									$\frac{853}{2160}$	$\frac{-7}{72\sqrt{2}}$	$\frac{37}{180}$
10										$\frac{3331}{8640}$	$\frac{-13}{144\sqrt{2}}$
11											$\frac{887}{2160}$

Table B.2c: Analytic expressions for the color magnetic matrix elements (δ–force)

ct_{ij}	$j=1$	2	3	4	5	6	7	8	9	10	11
$i=1$	$\frac{1}{12}$	$\frac{4}{15\sqrt{15}}$	$\frac{1}{6\sqrt{30}}$	$\frac{5}{12\sqrt{15}}$	$\frac{-7}{12\sqrt{30}}$	0	0	$\frac{-29}{90\sqrt{15}}$	$\frac{-1}{4\sqrt{30}}$	$\frac{257}{360\sqrt{15}}$	$\frac{29}{36\sqrt{30}}$
2		$\frac{74}{3375}$	$\frac{16}{1350\sqrt{2}}$	0	0	0	0	$\frac{133}{3375}$	$\frac{-7\sqrt{2}}{1800}$	$\frac{1}{125}$	$\frac{-1}{200\sqrt{2}}$
3			$\frac{2}{135}$	0	0	0	0	$\frac{-7\sqrt{2}}{1800}$	$\frac{-1}{45}$	$\frac{-1}{200\sqrt{2}}$	0
4				$\frac{-1}{108}$	$\frac{1}{54\sqrt{2}}$	$\frac{3}{5\sqrt{2}}$	$\frac{-1}{20}$	$\frac{-1}{90}$	$\frac{-1}{360\sqrt{2}}$	$\frac{1}{120}$	$\frac{-1}{144\sqrt{2}}$
5					$\frac{-11}{108}$	$\frac{-1}{20}$	$\frac{1}{5\sqrt{2}}$	$\frac{-1}{360\sqrt{2}}$	$\frac{7}{90}$	$\frac{-1}{144\sqrt{2}}$	$\frac{-1}{120}$
6						0	0	$\frac{1}{5\sqrt{2}}$	$\frac{-1}{60}$	0	0
7							0	$\frac{-1}{60}$	$\frac{4}{45\sqrt{2}}$	0	0
8								$\frac{-2717}{40500}$	$\frac{\sqrt{2}}{675}$	$\frac{-526}{3375}$	$\frac{109}{3600\sqrt{2}}$
9									$\frac{2771}{12960}$	$\frac{109}{3600\sqrt{2}}$	$\frac{37}{1620}$
10										$\frac{-3127}{10800}$	$\frac{11}{5400\sqrt{2}}$
11											$\frac{-103}{4320}$

Table B.2d: Analytic expressions for the gluon tensor matrix elements

p_{ij}	$j=1$	2	3	4	5	6	7	8	9	10	11
i=1	$\frac{1}{3}$	$\frac{-10}{3\sqrt{15}}$	$\frac{16}{3}\sqrt{\frac{2}{15}}$	$\frac{-10}{3\sqrt{15}}$	$\frac{16}{3}\sqrt{\frac{2}{15}}$	$-5\sqrt{\frac{2}{15}}$	$\frac{16}{\sqrt{15}}$	$\frac{25}{3\sqrt{15}}$	$\frac{-4}{3}\sqrt{\frac{2}{15}}$	$\frac{-55}{6\sqrt{15}}$	$\frac{47}{6}\sqrt{\frac{2}{15}}$
2		$\frac{-1}{3}$	$\frac{\sqrt{2}}{12}$	0	0	0	0	$\frac{7}{9}$	$\frac{2\sqrt{2}}{9}$	$\frac{-1}{6}$	0
3			$\frac{-1}{15}$	0	0	0	0	$\frac{2\sqrt{2}}{9}$	$\frac{-2}{45}$	0	$\frac{-1}{15}$
4				$\frac{1}{8}$	$\frac{-3}{4\sqrt{2}}$	$\frac{-1}{4\sqrt{2}}$	$\frac{3}{2}$	$\frac{-1}{72}$	$\frac{-11}{36\sqrt{2}}$	$\frac{-1}{12}$	$\frac{1}{6\sqrt{2}}$
5					$\frac{1}{10}$	$\frac{3}{2}$	$\frac{-1}{10\sqrt{2}}$	$\frac{-11}{36\sqrt{2}}$	$\frac{-1}{90}$	$\frac{1}{6\sqrt{2}}$	$\frac{-1}{5}$
6						$\frac{1}{4}$	$\frac{-1}{3\sqrt{2}}$	$\frac{-1}{4\sqrt{2}}$	$\frac{1}{2}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3}$
7							$\frac{1}{15}$	$\frac{1}{2}$	$\frac{-1}{5\sqrt{2}}$	$\frac{1}{3}$	$\frac{4}{15\sqrt{2}}$
8								$\frac{1}{72}$	$\frac{-11}{36\sqrt{2}}$	$\frac{1}{6}$	$\frac{1}{6\sqrt{2}}$
9									$\frac{1}{90}$	$\frac{1}{6\sqrt{2}}$	$\frac{1}{15}$
10										$\frac{1}{72}$	$\frac{-2}{9\sqrt{2}}$
11											$\frac{1}{45}$

Table B.3a: Analytic expressions for the pion exchange matrix elements (central part)

$p1_{ij}$	$j=1$	2	3	4	5	6	7	8	9	10	11
$i=1$	5	1	1	1	1	1	1	1	1	1	1
2		$\frac{29}{5}$	1	0	0	0	0	$\frac{1}{10}$	1	$\frac{77}{10}$	0
3			28	0	0	0	0	1	1	0	17
4				$\frac{69}{2}$	1	$\frac{21}{2}$	1	$\frac{77}{2}$	1	$\frac{7}{2}$	1
5					$\frac{87}{2}$	1	27	1	$\frac{71}{2}$	1	$\frac{3}{2}$
6						$\frac{33}{2}$	1	$\frac{7}{2}$	1	$\frac{35}{2}$	1
7							$\frac{113}{2}$	1	$\frac{9}{2}$	1	19
8								$\frac{1277}{10}$	1	$\frac{61}{10}$	1
9									$\frac{331}{2}$	1	14
10										$\frac{1127}{10}$	1
11											$\frac{259}{4}$

Table B.3b: p1 for the analytic expressions for the pion exchange matrix elements

$p3_{ij}$	$j=1$	2	3	4	5	6	7	8	9	10	11
$i=1$	-2	$\frac{-1}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$
2		$\frac{-11}{3}$	$\frac{-2}{3}$	0	0	0	0	$\frac{1}{3}$	$\frac{-2}{3}$	7	0
3			$\frac{-53}{3}$	0	0	0	0	$\frac{-2}{3}$	$\frac{4}{3}$	0	16
4				-9	$\frac{-2}{3}$	$\frac{-11}{3}$	$\frac{-2}{3}$	61	$\frac{-2}{3}$	$\frac{7}{3}$	$\frac{-2}{3}$
5					-7	$\frac{-2}{3}$	$\frac{-2}{3}$	$\frac{-2}{3}$	$\frac{97}{3}$	$\frac{-2}{3}$	$\frac{13}{9}$
6						$\frac{-49}{9}$	$\frac{-2}{3}$	$\frac{-11}{9}$	$\frac{-2}{3}$	$\frac{1}{3}$	$\frac{-2}{3}$
7							$\frac{-64}{3}$	$\frac{-2}{3}$	$\frac{-1}{9}$	$\frac{-2}{3}$	$\frac{-13}{6}$
8								$\frac{41}{3}$	$\frac{-2}{3}$	$\frac{19}{3}$	$\frac{-2}{3}$
9									$\frac{107}{3}$	$\frac{-2}{3}$	$\frac{41}{3}$
10										$\frac{-25}{3}$	$\frac{-2}{3}$
11											$\frac{-11}{3}$

Table B.3c: p3 for the analytic expressions for the pion exchange matrix elements

p5$_{ij}$	$j=1$	2	3	4	5	6	7	8	9	10	11
$i=1$	0	0	0	0	0	0	0	0	0	0	0
2		$\frac{11}{75}$	$\frac{1}{15}$	0	0	0	0	$\frac{-11}{150}$	$\frac{1}{15}$	$\frac{11}{50}$	0
3			$\frac{2}{3}$	0	0	0	0	$\frac{1}{15}$	$\frac{-1}{3}$	0	1
4				$\frac{11}{10}$	$\frac{1}{15}$	$\frac{11}{10}$	$\frac{1}{15}$	$\frac{-11}{6}$	$\frac{1}{15}$	$\frac{11}{30}$	$\frac{1}{15}$
5					$\frac{1}{2}$	$\frac{1}{15}$	1	$\frac{1}{15}$	$\frac{19}{6}$	$\frac{1}{15}$	$\frac{1}{18}$
6						$\frac{11}{18}$	$\frac{1}{15}$	$\frac{11}{30}$	$\frac{1}{15}$	$\frac{11}{30}$	$\frac{1}{15}$
7							$\frac{17}{6}$	$\frac{1}{15}$	$\frac{1}{6}$	$\frac{1}{15}$	$\frac{7}{6}$
8								$\frac{473}{150}$	$\frac{1}{15}$	$\frac{-11}{150}$	$\frac{1}{15}$
9									$\frac{-1}{6}$	$\frac{1}{15}$	$\frac{1}{3}$
10										$\frac{803}{150}$	$\frac{1}{15}$
11											$\frac{41}{12}$

Table B.3d: p5 for the analytic expressions for the pion exchange matrix elements

(ij)	s_{ij}	$s1_{ij}$	$s3_{ij}$	$s5_{ij}$
(11)	1	12	1	0
$(22), (44), (66), (88), (10,10)$	$\frac{1}{3}$	29	$\frac{5}{3}$	$\frac{11}{15}$
$(33), (55), (77), (99), (11,11)$	$\frac{1}{3}$	28	$\frac{7}{3}$	$\frac{2}{3}$
$(23), (45), (67), (89), (10,11)$	$\frac{-5}{6\sqrt{2}}$	1	$\frac{-2}{3}$	$\frac{1}{15}$

Table B.4: Analytic expressions for the non-zero sigma exchange matrix elements

41